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Near-horizon limit of the charged BTZ black hole and AdS_2 quantum gravity

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Abstract: We show that the 3D charged Banados-Teitelboim-Zanelli (BTZ) black hole solution interpolates between two different 2D AdS spacetimes: a near-extremal, near-horizon AdS_2 geometry with constant dilaton and U(1) field and an asymptotic AdS_2 geometry with a linear dilaton. Thus, the charged BTZ black hole can be considered as interpolating between the two different formulations proposed until now for AdS_2 quantum gravity. In both cases the theory is the chiral half of a 2D CFT and describes, respectively, Brown-Hennaux-like boundary deformations and near-horizon excitations. The central charge c_{as} of the asymptotic CFT is determined by 3D Newton constant G and the AdS length l, $c_{as} = 3l/G$, whereas that of the near-horizon CFT also depends on the U(1) charge Q, $c_{nh} \propto lQ/\sqrt{G}$.

Keywords: AdS-CFT Correspondence, Conformal and W Symmetry, Black Holes.

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Contents

1. Introduction

Quantum gravity in low-dimensional anti-de Sitter(AdS) spacetime has features that make it peculiar with respect to the higher-dimensional cases. For $d = 2, 3$ the theory is a conformal field theory (CFT) describing (Brown-Hennaux-like) boundary deformations and has a central charge determined completely by Newton constant and the AdS length [1–4]. Conversely, in $d > 4$, quantum gravity in AdS spacetimes should admit a near-horizon description in terms of BPS solitons and D-brane excitations, whose low-energy limit is an U(N) gauge theory [5–7].

The difference between these two descriptions is particularly evident in their application for computing the entropy of non-perturbative gravitational configurations such as black holes, black branes and BPS states. Brown-Hennaux-like boundary excitations have been used with success to give a microscopically explanation to entropy of the BTZ black hole and of two-dimensional (2D) AdS (AdS_2) black holes [8, 2]. On the other hand, D-brane excitations account correctly for the entropy of extremal and near-extremal Reissner-Nordstrom black holes in higher dimensions [5].

Moreover, the status of the AdS_2/CFT_1 correspondence [2, 3, 9–13] remains still enigmatic. The dual CFT_1 has been identified both as a conformal mechanics and as a chiral half of a 2D CFT. Progress towards a better understanding of the relationship between low- and higher-dimensional AdS/CFT correspondence has been achieved in ref. [11]. It has been shown that quantum gravity on AdS_2 with constant electromagnetic (EM) field and dilaton can be described by the chiral half of a twisted CFT with central charge proportional to the square of the EM field.

On the other hand, there is another formulation of AdS_2 quantum gravity, which uses Brown-Hennaux-like boundary states in a 2D AdS spacetime endowed with a linear dilaton [2]. Also in this case the Hilbert space of the theory falls into the representation of a chiral half of a CFT, but the central charge is proportional to the inverse of 2D newton constant. The results of ref. [11] raise the question about the relationship between the two different realizations of AdS_2 quantum gravity.

In this paper we show that a bridge between these two formulations is three-dimensional (3D) AdS-Maxwell gravity. We find that the charged BTZ black hole admits two limiting regimes (near-horizon and asymptotic) in which the black hole is described by a 2D Maxwell-dilaton theory of gravity. In the near-horizon, near-extremal regime the black hole is described by AdS_2 with a constant dilaton and $\mathrm{U}(1)$ field. In the asymptotic regime the BTZ black hole is described by AdS_2 with a linear dilaton background and $\mathrm{U}(1)$ field strength $F_{tr} = Q/r$.

Both regimes are in correspondence with a CFT_1, which can be thought as the chiral half of a 2D CFT. The central charge of the near-horizon CFT is proportional to the electric charge Q of the BTZ black hole $c_{nh} = (3k/4)\sqrt{\pi/G}lQ$ where k is the level of the $\mathrm{U}(1)$ current. The central charge of the asymptotic CFT is determined completely by 3D Newton constant G and the AdS length l: $c_{as} = 3l/G$.

We can therefore think of the charged BTZ black hole as an interpolating solution between the near-extremal, near-horizon behavior typical of BPS-like solutions in higher dimensions (e.g. Reissner-Nordstrom black hole solutions in four and five dimensions) and the asymptotic behavior typical of Brown-Henneaux-like states.

This paper is organized as follows. In section 2 we review briefly the features of the charged BTZ black hole. In section 3 we investigate the two limiting regimes, namely the near-horizon limit and the asymptotic $r \to \infty$ limit. In section 4 we describe the dimensional reduction from three to two spacetime dimensions. In section 5 we investigate the CFTs that describe the two different regimes and calculate the corresponding central charges. Finally, in section 6 we present our conclusions.

2. The charged BTZ black hole

The charged BTZ black hole solutions are a generalization of the well-known black hole solutions in (2+1) spacetime dimensions derived by Banados, Teitelboim and Zanelli [14, 15].

They are derived from a three-dimensional theory of gravity

$$I = \frac{1}{16\pi G}\int d^3x\sqrt{-g_{(3)}}\left(R + \frac{2}{l^2} - 4\pi G F_{\mu\nu}F^{\mu\nu}\right), \tag{2.1}$$

where G is 3D Newton constant, $\frac{1}{l^2}$ is the cosmological constant (l is the AdS-length) and $F_{\mu\nu}$ is the electromagnetic field strength. We consider the BTZ black hole with zero angular momentum and use the conventions of ref. [16].

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Electrically charged black hole solutions of the action (2.1) are characterized by the U(1) Maxwell field [14, 17],

$$F_{tr} = \frac{Q}{r}, \tag{2.2}$$

where Q is the electric charge. The 3D line element is given by

$$ds_3^2 = -f(r)dt^2 + f^{-1}dr^2 + r^2 d\theta^2, \tag{2.3}$$

with metric function:

$$f(r) = -8GM + \frac{r^2}{l^2} - 8\pi GQ^2 \ln\left(\frac{r}{w}\right), \tag{2.4}$$

where M, w are constants and $-\infty < t < +\infty$, $0 \le r < +\infty$, $0 \le \theta \le 2\pi$. Although the solution depends only on a single integration constant (w can be absorbed by a redefinition of M), we use both M and w to parametrize the solution (2.3). We will come back to this point later on this section. Notice that if we require that for $Q = 0$, M reduces to the Arnowitt-Deser-Misner (ADM) mass of the uncharged BTZ black hole we must keep both w and M as independent parameters.

The black hole has one inner (r_-) and outer (r_+) , one or no horizons depending on whether

$$\Delta = 8GM - 4\pi GQ^2\left[1 - 2\ln\left(\frac{2Ql}{w}\sqrt{\pi G}\right)\right] \tag{2.5}$$

is greater than, equal to or less than zero, respectively. Although these solutions for $r \to \infty$ are asymptotically AdS, they have a power-law curvature singularity at $r = 0$, where $R \sim (8\pi GQ^2)/r^2$. This $r \to 0$ behavior of the charged BTZ black hole has to be compared with that of the uncharged one, for which $r = 0$ represents just a singularity of the causal structure.

The Hawking temperature T_H associated with the outer black hole horizon is

$$T_H = \frac{r_+}{2\pi l^2} - \frac{2GQ^2}{r_+}. \tag{2.6}$$

According to the Bekenstein-Hawking formula, the thermodynamic entropy of a black hole is proportional to the area A of the outer event horizon, $S = \frac{A}{4G}$. For the charged BTZ black hole we have

$$S = \frac{\pi r_+}{2G} = \frac{\pi l}{G}\sqrt{2GM + 2\pi GQ^2 \ln\left(\frac{r_+}{w}\right)}. \tag{2.7}$$

Owing to the presence of the $\ln r$ term in eq. (2.4) the definition of the mass of the solution is problematic. By varying the action (2.1) one gets a surface term which diverges logarithmically for $r \to \infty$ [17]. This divergence can be handled by enclosing the system in a circle of radius r_0, which in terms of the dual CFT has to be interpreted as a UV cutoff. Writing the metric function (2.4) as $f(r) = r^2/l^2 - 8GM_0(r_0, w) - 8\pi GQ^2 \ln(r/r_0)$, one can define a regularized mass [17]

$$M_0(r_0, w) = M + \pi Q^2 \ln\left(\frac{r_0}{w}\right). \tag{2.8}$$

In the limit $r \to \infty$, one takes also $r_0 \to \infty$ keeping the ratio $r/r_0 = 1$. This procedure allows for the definition of a regularized black hole mass $M_0(r_0, w)$, which has to be interpreted as the total energy (electromagnetic and gravitational) inside the circle of radius r_0. Physical insight on this renormalization procedure can be obtained if one thinks of w, in the spirit of the Wilson renormalization group, as a running scale. Basically one has two options: *a*) one takes M fixed and the metric and the entropy scale-dependent, *b*) the metric and the entropy are w-invariant and M runs with w.[1]

Option *a*) corresponds to the renormalization prescription of ref. [17], where eq. (2.8) is used to identify M with the total mass of the solution (the sum of the energy inside the circle of radius r_0 and the electrostatic energy outside it). The entropy change is due to the new degrees of freedom that are excited when w rises. The drawback of this prescription is that the position of the black hole horizon r_+ (the IR scale in the dual CFT) is not held fixed but changes with w.

Because we want to keep r_+ fixed (this seems necessary in order to give a microscopic interpretation to the black hole entropy) we will use prescription *b*). The IR scale is held fixed when $w \to \lambda w$ and the running of the logarithmic term is compensated by the change of M, $M \to M + \pi Q^2 \ln \lambda$, so that both the metric function f and M_0 do not change. This behavior share some analogy with the UV generation of bigger and bigger flux for the 5-form in the context of singular IIB supergravity solutions describing fractional 3-branes [18].

As a consequence of the w-invariance of f, S, M_0, w and M_0 can be arbitrarily chosen. The most natural physical choice is to first fix the energy scale in terms of the AdS length, $w = l$, then fix $M_0(r_0)$ to its horizon value by setting $r_0 = r_+$. According to this choice, in the following we will set $w = l$ in eqs. (2.4), (2.5), (2.7). The invariant mass of the solution, which has to be identified with the conserved charge associated with the time translation invariance is therefore

$$M_0(r_+) = M + \pi Q^2 \ln\left(\frac{r_+}{l}\right). \tag{2.9}$$

3. The near-horizon limit

We are interested in the near-horizon, near-extremal behavior of the solution (2.3). It is well known that in this regime asymptotically flat charged black holes in $d \geq 4$ dimensions are described by a $AdS_2 \times S^{d-2}$ geometry, i.e a Bertotti-Robinson spacetime. The flux of the EM field stabilize the radius of the transverse sphere, so that in the near-horizon, near-extremal limit it becomes constant and given in terms of the EM charge. Let us show that this is also the case for the charged BTZ black hole.

The extremal limit $r_+ = r_- = \gamma$ of the BTZ black hole is characterized by $\Delta = 0$ in eq. (2.5), so that γ is a double zero of the metric function (2.4):

$$\gamma = 2\sqrt{\pi G} Q l. \tag{3.1}$$

In order to describe the near-horizon, near-extremal limit of our three-dimensional solution we perform a translation of the radial coordinate r,

$$r = \gamma + x, \tag{3.2}$$

[1]We thank the anonymous referee of JHEP for this remark

and expand both the metric function (2.3) and U(1) field (2.2) in powers of x. We get after some manipulations

$$f(x) = \frac{2}{l^2}x^2 - 8G\Delta M + O(x^3), \quad F_{tx} = \frac{1}{2\sqrt{\pi G}\, l} + O(x), \tag{3.3}$$

where $\Delta M = M - M(\gamma) = M - \pi Q^2(\frac{1}{2} - \ln(2Q\sqrt{\pi G}))$ is the mass above extremality. In the near-horizon, near-extremal limit the topology of the 3D solution factorize as $AdS_2 \times S^1$ and the geometry becomes,

$$ds^2_{(3)} = -\left(\frac{2}{l^2}x^2 - 8G\Delta M\right)dt^2 + \left(\frac{2}{l^2}x^2 - 8G\Delta M\right)^{-1}dx^2 + \gamma^2 d\theta^2, \quad F_{tx} = \frac{1}{2\sqrt{\pi G}l}. \tag{3.4}$$

The mass of the excitations above extremality can be also expressed in terms of $\Delta r_+ = r_+ - \gamma$. Up to order three in Δr_+ we have

$$\Delta M = \frac{\Delta r_+^2}{4Gl^2}. \tag{3.5}$$

The near-horizon, extremal limit of the 3D charged AdS black hole is therefore very similar to that of its higher-dimensional, asymptotically flat, cousins such as the Reissner-Nordstrom solution in four and five dimensions. In particular, our 3D solution shares with them the thermodynamical behavior. From eqs. (2.6). (2.7), (3.2) one easily finds that the extremal charged BTZ black hole is a state of zero temperature and constant entropy

$$S_{(ext)} = \frac{\pi\gamma}{2G} = \pi\sqrt{\frac{\pi}{G}}Ql. \tag{3.6}$$

For small excitations near extremality we get using (3.5)

$$S_{ne} = \frac{\pi\gamma}{2G} + \pi\frac{\Delta r_+}{2G} = \frac{\pi\gamma}{2G} + \pi l\sqrt{\frac{\Delta M}{G}}. \tag{3.7}$$

3.1 The asymptotic $r \to \infty$ limit

It is also interesting to discuss briefly the asymptotic $r \to \infty$ limiting case of the 3D solution (2.3) and its relationship with the near-horizon solution (3.4). In the $r \to \infty$ limit the metric describes 3D AdS spacetime, whereas the U(1) field goes to zero as $1/r$. As we shall see in detail in the next section also in this regime the 3D solution admits an effective description in terms of AdS_2 endowed with a linear varying dilaton. The dilaton parametrizes the radius of the transverse one-sphere, which in the $r \to \infty$ limit diverges. We can therefore think of the full charged BTZ solution (2.3) as a 3D spacetime interpolating between two regimes admitting an effective description in terms of AdS_2.

4. Dimensional reduction of the charged BTZ black hole

The two limiting regimes of the BTZ black hole can be described by an effective 2D Maxwell-Dilaton gravity model. In order to find this 2D description, we parametrize the radius of the S^1 sphere in the 3D solution (2.3) with a scalar field (the dilaton) ϕ:

$$ds^2_{(3)} = ds^2_{(2)} + l^2\phi^2 d\theta^2. \tag{4.1}$$

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where $ds_{(2)}$ is the line element of the 2D sections of the 3D spacetime covered by the (t,r) coordinates and ϕ is a function of t,r only. We will consider only electric configurations for the 3D maxwell field, i.e we use for $F_{\mu\nu}$ the ansatz

$$F_{t\theta} = F_{r\theta} = 0. \tag{4.2}$$

Using eqs. (4.1) and (4.2) into the 3D action (2.1) one obtains, after defining the rescaled dilaton $\eta = (l/4G)\phi$, the dimensionally reduced 2D action,

$$I = \frac{1}{2}\int d^2x\sqrt{-g}\,\eta\left(R + \frac{2}{l^2} - 4\pi G F^2\right). \tag{4.3}$$

The field equation stemming from the action (4.3) are

$$\begin{aligned} R + \frac{2}{l^2} - 4\pi G F^2 &= 0 \\ \nabla_\mu(\eta F^{\mu\nu}) &= 0 \\ -\nabla_\mu\nabla_\nu\eta + \left[\nabla^2\eta - \frac{\eta}{l^2} + 2\pi G\eta F^2\right] g_{\mu\nu} &= 8\pi G\eta F_{\mu\beta}F^{\beta}_{\nu}. \end{aligned} \tag{4.4}$$

It is important to notice that the field equations are invariant under rescaling of the dilaton by a constant. This constant mode of the dilaton is therefore classically undetermined but it can be fixed by matching the 2D with the 3D solution.

The field equations (4.4) admit two classes of solutions whose metric part is always a 2D AdS spacetime: 1) AdS_2 with linear dilaton and with electric field which vanishes asymptotically (corresponding to the asymptotic $r \to \infty$ regime of the charged BTZ black hole); 2) AdS_2 with constant dilaton and electric field (corresponding to the near-horizon limit of the BTZ black hole). Let us discuss separately these solutions.

4.1 AdS_2 with a linear dilaton

This solution of the field eqs. (4.4) is just the 3D solution (2.3) written in a two-dimensional form,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2, \quad F_{\mu\nu} = \frac{Q}{r}\epsilon_{\mu\nu}, \quad \eta = \bar{\eta}_0\frac{r}{l} \tag{4.5}$$

where $f(r)$ has exactly the same form as given by eq. (2.4), Q is the electric charge and $\bar{\eta}_0$ is an integration constant related to the scale symmetry of the 2D field equations. The integration constants appearing in eq. (4.5) (thus defining the physical parameters of the 2D black hole) can be easily identified in terms of the physical parameters of the BTZ black hole. The charge Q and mass M of the 2D black hole are the same as those of the BTZ black hole. The constant $\bar{\eta}_0$ is determined by the ansatz (4.1),

$$\bar{\eta}_0 = \frac{l}{4G}. \tag{4.6}$$

With this identification also the temperature and entropy of the 2D black hole match exactly those for the 3D black hole given by eqs. (2.6) and (2.7). For instance, the entropy of the 2D black hole is determined by the value of the dilaton on the horizon,

$$S = 2\pi\eta_{\text{horizon}}, \tag{4.7}$$

which after using eqs. (4.5) and (4.6) reproduces exactly eq. (2.7).

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4.2 AdS_2 with constant dilaton and electric field

One can easily realize that the field equations (4.4) admit a solution describing AdS_2 with constant dilaton and electric field. The constant value of the dilaton, which is not fixed by the 2D field equations, is determined by the ansatz (4.1),

$$\eta_0 = \frac{l}{2}\sqrt{\frac{\pi}{G}}Q. \tag{4.8}$$

In order to have the usual normalization of the electric field and to make contact with the model investigated in ref. [11], it is necessary to perform a Weyl transformation of the metric and a rescaling of the U(1) field strength:

$$g_{\mu\nu} = \frac{\eta}{\eta_0}\bar{g}_{\mu\nu}, \quad F_{\mu\nu} = \frac{l}{2\sqrt{2\pi G\eta_0}}\bar{F}_{\mu\nu}. \tag{4.9}$$

After this transformation the 2D action (4.3), modulo total derivatives, becomes

$$I = \frac{1}{2}\int d^2x\sqrt{-\bar{g}}\left[\eta\left(R(\bar{g}) + \frac{(\partial\eta)^2}{\eta} + \frac{2\eta}{l^2\eta_0}\right) - \frac{l^2}{2}\bar{F}^2\right]. \tag{4.10}$$

The field equations stemming from this action allow for a solution describing AdS_2 with constant dilaton and electric field, which is the dimensional reduction of the near-horizon solution (3.4)

$$\begin{aligned} ds^2 &= -(\frac{2}{l^2}x^2 - a^2)dt^2 + (\frac{2}{l^2}x^2 - a^2)^{-1}dx^2, \quad \bar{F}_{\mu\nu} = 2E\epsilon_{\mu\nu}, \\ \eta &= 2l^4E^2, \quad E^2 = \frac{1}{4l^3}\sqrt{\frac{\pi}{G}}Q, \end{aligned} \tag{4.11}$$

where we have used eq. (4.8) and $a^2 = 8G\Delta M$.

Following ref. [11] we can linearize the term quadratic in the U(1) field strength by introducing in the action an auxiliary field h,

$$I = \frac{1}{2}\int d^2x\sqrt{-\bar{g}}\left[\eta\left(R(\bar{g}) + \frac{(\partial\eta)^2}{\eta} + \frac{2\eta}{l^2\eta_0}\right) - \frac{h^2}{l^2} + h\epsilon^{\mu\nu}\bar{F}_{\mu\nu}\right]. \tag{4.12}$$

The field equations for h give

$$h = \frac{l^2}{2}\epsilon^{\mu\nu}\bar{F}_{\mu\nu} = -2El^2. \tag{4.13}$$

5. Conformal symmetry and central charges

In view of the AdS/CFT correspondence, the existence of two limiting AdS_2 configurations for the charged BTZ black hole imply the duality of the gravitational configuration with two different CFTs. Both CFTs have been already investigated in the literature and in both of them the conformal transformations appear as a subgroup of the 2D diffeomorphisms. However, they differ in the way the central charge of the CFT is generated. The CFT associated with the $r \to \infty$ limit, corresponding to AdS_2 with a linear dilaton has been investigated in ref. [2]. In this case the central charge of the CFT is generated by the breaking of the $SL(2, R)$ isometry of the AdS_2 background due to the non-constant dilaton [19].

The CFT associated with the near-horizon limit, corresponding to AdS_2 with a constant electric and dilaton field has been investigated in ref. [11]. In this case the central charge of the CFT is generated by the boundary conditions for the EM vector potential. We will discuss the two cases separately.

5.1 The $r \to \infty$ asymptotic CFT

In this case the conformal algebra is generated by the group of asymptotic symmetries (ASG) of AdS_2 along the lines of ref. [2, 16]. The calculations of refs. [2] can be easily extended to the theory described by the action (4.3). The only difference is the presence of the U(1) field, which however, as explained in Ref [16] for the case of 3D gravity, does not change neither the conformal algebra, which is always given by a chiral half of the Virasoro algebra, nor the value of the central charge.

The $r \to \infty$ boundary conditions for the fields, which are invariant under 2D diffeomorphisms generated by killing vectors $\chi^t = l\epsilon(t) + \mathcal{O}(1/r^2), \chi^r = -lr\dot{\epsilon}(t) + \mathcal{O}(1/r)$ are

$$
\begin{aligned}
g_{tt} &= -\frac{r^2}{l^2} + \mathcal{O}(\ln r), \quad g_{tr} = \mathcal{O}\left(\frac{1}{r^3}\right), \\
g_{rr} &= \frac{l^2}{r^2} + \mathcal{O}\left(\frac{\ln r}{r^4}\right), \quad \eta = \mathcal{O}(r), \quad F_{tr} = \mathcal{O}\left(\frac{1}{r}\right).
\end{aligned} \tag{5.1}
$$

Notice that we allow for deformations of the dilaton and EM field that are of the same order of the background solution (4.5). Although the boundary conditions (5.1) are invariant under the action of the asymptotic symmetry group, the classical solution is not. The linear dilaton and the Q/r EM field break the isometry group of AdS_2. The breaking of the isometry group due to the linear dilaton background produces a nonvanishing central charge in the conformal algebra [19]. Conversely, the EM field does not contribute to the boundary charges, but only enters in the renormalization of the L_0 Virasoro operator [16].

The generators of the conformal diffeomorphisms close in the Virasoro algebra

$$
[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n\,0}. \tag{5.2}
$$

The central charge c can be computed using a canonical realization of the ASG along the lines of refs. [2, 10, 16]. One has

$$
c = 12\bar{\eta}_0 = \frac{3l}{G}. \tag{5.3}
$$

where we have used eq. (4.6).

The eigenvalue l_0 of the Virasoro operator L_0 has to identified with the conserved charge associated with the operator generating time-translations. It is therefore given by the mass $M_0(r_+)$ of eq. (2.9),

$$
l_0 = lM_0(r_+) = l\left[M + \pi Q^2 \ln\left(\frac{r_+}{l}\right)\right] \tag{5.4}
$$

5.2 The near-horizon CFT

The 2D action (4.12) can be recast in the form of a twisted 2D CFT in which a central term in the Virasoro algebra is generated by boundary conditions for the U(1) vector potential A_μ, along the lines of ref. [11].

Using a conformal and Lorentz gauge respectively, we fix the diffeomorphisms and U(1) gauge freedom,

$$ds^2 = -e^{2\rho} dx^+ dx^-, \quad \partial_\mu A^\mu = 0, \tag{5.5}$$

the action (4.12) becomes up to total derivatives

$$I = \frac{1}{2} \int d^2x \left(-4\partial_- \eta \partial_+ \rho + \frac{\eta}{l^2 \eta_0} + 2\frac{\partial_- \eta \partial_+ \eta}{\eta} - \frac{h^2}{2l^2} + 4\partial_- h \partial_+ a \right), \tag{5.6}$$

where we have used the fact that in the gauge (5.5) A_μ can be given in terms of a scalar a, $A^\mu = \epsilon^{\mu\nu}\partial_\nu a$.

As usual for gauge-fixing the classical field equations stemming from the action (5.6) must be supported by constraints,

$$T_{\pm\pm} = \frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\pm\pm}} = -2\partial_\pm \eta \partial_\pm \rho + \partial_\pm \partial_\pm \eta - \eta^{-1} \partial_\pm \eta \partial_\pm \eta + 2\partial_\pm h \partial_\pm a = 0, \tag{5.7}$$

$$J_\pm = 2\frac{\delta I}{\delta A^\pm} = \pm 2\partial_\pm h = 0. \tag{5.8}$$

The stress-energy tensor $T_{\pm\pm}$ and the U(1) current $J_\pm$ are (classically) holomorphic conserved and generate, respectively, residual conformal diffeomorphisms and gauge transformations.

In the conformal gauge the vacuum AdS_2 solution (4.11) becomes

$$ds^2 = -2l^2 \frac{dx^+ dx^-}{(x^+ - x^-)^2}, \quad A_\pm = \frac{El^2}{2\sigma}, \tag{5.9}$$

where $\sigma = (1/2)(x^+ - x^-)$ and h, η, E are given by eqs. (4.11), (4.13).

Because the dilaton is constant one naively expects that we are dealing with pure 2D quantum gravity, which is known to be described by a CFT with vanishing central charge [20]. However, it has been shown in [11] that the boundary conditions for the U(1) vector potential at the $\sigma = 0$ conformal boundary of AdS_2, $A_\sigma|_{\sigma=0} = 0$, is not preserved by conformal diffeomorphisms generated by $\chi^+(x^+)$ and $\chi^-(x^-)$. It must be accompanied by a gauge transformation $\omega^+(x^+) + \omega^-(x^-)$, which in the case under consideration is given by

$$\omega^\pm = \mp \frac{l^2 E}{2} \partial_\pm \chi^\pm. \tag{5.10}$$

Moreover, the requirement the boundary remains at $\sigma = 0$ determines a chiral half of the conformal diffeomorphisms in terms of the second half. The resulting conformal symmetry can be realized using Dirac brackets. Conformal transformations are generated by the improved stress-energy tensor

$$\tilde{T}_{--} = T_{--} - \frac{El^2}{2} \partial_- J_-. \tag{5.11}$$

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Expanding in Laurent modes and using the transformation law of the improved stress-energy tensor

$$\delta_\chi \tilde{T}_{--} = \chi^- \partial_- \tilde{T}_{--} + 2\partial_- \chi^- \tilde{T}_{--} + \frac{c}{12}\partial_-^3 \chi^-, \tag{5.12}$$

where we allow for the existence of an anomalous term, one finds that the operators $\tilde{L}$ span the Virasoro algebra (5.2). The transformation law of the original T_{--} is anomaly-free, but that of the current J_- may have an anomalous term proportional to its level k [11],

$$\delta_\omega J_- = k\partial_- \omega^-. \tag{5.13}$$

This allows us to compute the central charge c of the Virasoro algebra,

$$c = 3kE^2 l^4 = \frac{3}{4} k \sqrt{\frac{\pi}{G}}\, lQ. \tag{5.14}$$

6. Conclusion

Using the results of the previous section we can reproduce the entropy of the 2D AdS black hole (and the entropy of the charged BTZ black hole) by calculating the density of states $\rho(l_0)$ of the CFT with a given eigenvalue l_0. In the semiclassical limit $c \gg 1$ and for large l_0 we have Cardy formula,

$$S = \ln \rho(l_0) = 2\pi \sqrt{\frac{c l_0}{6}} \tag{6.1}$$

Using Eqs (5.3) and (5.4) we reproduce exactly the black hole entropy (2.7).

In principle, one should also be able to reproduce the entropy of the near-extremal black hole (3.7) using a similar procedure for the near-horizon twisted CFT. Obviously to calculate the density of states we have to use in the cardy formula (6.1) the eigenvalues of the twisted operator $\tilde{L}_0$ (the Hamiltonian of the twisted CFT), $\tilde{l}_0$, instead of that of the untwisted operator L_0. Calculation of $\tilde{l}_0$ requires careful analysis of the CFT spectrum and detailed knowledge of the effect of the twisting on the Hilbert space of the 2D CFT.

It is important to stress that the eigenvalue of the L_0 operator, the Hamiltonian of the untwisted CFT, must be zero. This is a consequence of the fact that from the purely two-dimensional point of view the solution (4.11) has zero mass. The mass can be easily calculated using the usual ADM procedure, i.e considering linear perturbations near the AdS_2 background (the solution (4.11) with $a^2 = 0$). It is well known that any 2D, asymptotically AdS, solution of the form (4.11) is locally equivalent to the AdS_2 background, i.e a^2 can be gauged away by a 2D diffeomorphism. On the other hand, being the dilaton and EM field constant, there is no global obstruction to prevent maximal extension of the spacetime beyond the horizon to recover the AdS_2 background. It follows immediately that the ADM mass of the solution (4.11) must vanish.[2]

[2]This is not the case for the 2D solution (4.5), which describes the AdS_2 black hole endowed with a linear dilaton. The dilaton is a non-constant scalar and its inverse gives 2D Newton constant. The Newton constant singularity at $r = 0$ has to be considered as a spacetime singularity that represents a global obstruction to the maximal extension of the spacetime [21]. The ADM mass of the solution is therefore non zero and given by M.

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The vanishing of the mass of the solution (4.11) is related with the impossibility of having at the classical level non-singular finite energy excitations of $AdS_2 \times S^2$ [22]. The fact that the mass of the 2D solution is zero does not imply that the 2D solutions (4.11) cannot have an associated thermodynamical description nor that the 3D solutions, of which they are the dimensional reduction, must also have zero mass. Two-dimensional AdS solutions with constant scalars typically emerge as the dimensional reduction of the near-horizon regime of extremal charged black holes in higher dimensions. The extremal limit saturates some BPS bound so that the mass of the extremal solution is different from zero. Although the mass of the 2D solution is zero one can formulate a thermodynamical description using the entropy function formalism [23]. Further insight in the thermodynamical behavior of these objects can also be gained using the attractor mechanism [24, 25].

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